Math 672: Projective Geometry I
Homework 1

Hand in between 2 and 5 inclusive of the problems below; mark at the top of your page which problem numbers you are handing in. Write full proofs for each of your solutions. Your score will be as a percentage of those you hand in. To submit your solutions, either type on Overleaf/LaTeX or handwrite and take pictures for file uploads, and upload them to the Canvas course under the appropriate Assignment.

Problems

1. Describe a projective transformation that takes the circle $x^2 + y^2 = z^2$ to the hyperbola $xy = z^2$.
2. Prove that an invertible projective transformation of $\mathbb{P}^1$, over any field of characteristic 0, is uniquely determined by where it sends three distinct points. (Recall that such a transformation is of the form $(x : y) \mapsto (ax + by : cx + dy)$, and we can consider the three points $(x : y)$ to be $(0 : 1)$, $(1 : 0)$, $(1 : 1)$.)