

# Math 601 - Advanced Combinatorics I - Fall 2024

## Basic Information

**Instructor:** Maria Gillespie, Maria.Gillespie@colostate.edu

**Office:** 125 Weber Building

**Course web page:** See the Canvas page for most announcements. Homeworks will also be posted at:

<http://mathematicalgemstones.com/maria/Math601Fall24.php>

**Class time and location:** MWF 1:00 pm, Engineering building room B-4

**Office hours:** Wednesday 2–4 pm

**Books:** There are **no required books**; coming to lecture (or catching up via recorded videos) will be sufficient for learning all material. Some helpful references are:

- *Representation Theory*, by Fulton and Harris
- *Introduction to Quantum Groups and Crystal Bases*, by Hong and Kang
- *Crystal Bases*, by Bump and Schilling
- *The Symmetric Group*, by Bruce Sagan
- *Combinatorics of Coxeter Groups* by Björner and Brenti
- *Young tableaux*, by William Fulton

**Prerequisites:** Familiarity with the basics of group theory and symmetric functions is helpful.

## Course policies

The grading for this class will be entirely homework based. Two of the homeworks will be long and cumulative in place of exams.

**Homework** will be due according to the schedule. Please submit your homework as a single typed or scanned PDF file. There will be two types of homeworks: **ordinary homeworks** and **cumulative homeworks**. Which type you will get on each week will vary (see the schedule below).

- **Ordinary homework format:** Each homework problem will be assigned a number of points based on difficulty. A 1 or 2 point problem is a routine exercise. A 3, 4, or 5 point problem is harder and may involve some clever thinking. Problems worth 6

points or over are very hard, and problems worth 10 or more are either open problems or recently published results. (Partial credit will be given for strong insights towards a problem.)

The points you earn are cumulative, and each homework is graded out of a maximum of 15 points. Your score on the homework will be

$$\min(T, 15)$$

where  $T$  is the total number of points of the problems you handed in correct solutions to. *Make sure you clearly indicate which problems you are handing in and what their point values are!*

You may hand in a set of problems whose total score is greater than 15 **if and only if** removing any one of the problems will make the total less than 15.

Collaboration is permitted, but you must list all collaborators on a problem's solution at the top of the page.

- **Cumulative homework format:** These should be thought of as small take-home tests. Collaboration is not permitted, though you may talk with your peers about other homework problems from the weekly homeworks in order to gain more understanding while working on the cumulative homework. All problems on these homeworks must be solved to receive full credit.

**Late homework policy:** There will be 10 percentage points deducted from your homework score for each day that you hand it in late, with a minimum of 0 points. If you hand in a homework, such as the final homework, after I have already submitted grades, you will get a 0 for that assignment.

**Grading:** Your lowest homework grade (calculated as a percentage, across homeworks of both types together) will be dropped to account for possible emergencies or illness. Otherwise your final grade is the weighted average of all your homework grades.

## Tentative Schedule

- Week 1: (Aug 19, 21, 23) Introduction: Lie groups, Lie algebras, the exponential map
- Week 2: (Aug 26, 28, 30) Representations of  $GL_n$ , Schur-Weyl duality
  - Homework 1 due Aug 30
- Week 3: (Sep 4,6) Representation theory of  $\mathfrak{sl}_2$
- Week 4: (Sep 9, 11, 13) Tensor product diagrams, ballot words in two letters
  - Homework 2 due Sep 13

- Week 5: (Sep 16, 18, 20) Representation theory of  $\mathfrak{sl}_3$ , weight lattice
- Week 6: (Sep 23, 25, 27)  $\mathfrak{sl}_3$  - tensor products, tableaux crystals, Schur function characters
  - Homework 3 due Sept 27
- Week 7: (Sept 30, Oct 2, 4) Representations of  $\mathfrak{sl}_n$ , Stembridge axioms for type A, general LR rule
  - **Cumulative Homework A due Oct 4**
- Week 8: (Oct 7, 9, 11) Classification of semisimple Lie algebras by root systems, dynkin diagrams
  - Homework 4 due Oct 11
- Week 9: (Oct 14, 16, 18) General crystal base theory, Stembridge axioms
- Week 10: (Oct 21, 23, 25) Weyl groups and Coxeter groups in other Lie types
  - Homework 5 due Oct 25
- Week 11: (Oct 28, 30, Nov 1) Springer theory in type A
- Week 12: (Nov 4, 6, 8) Hall-Littlewood polynomials and Springer theory in other Lie types
  - Homework 6 due Nov 8
- Week 13: (Nov 11, 13, 15) Diagonal coinvariants and Shuffle Theorem, affine flag variety
- Week 14: (Nov 18, 20, 22) Hessenberg varieties
  - **Cumulative Homework B due Nov 22**
- Week 15: (Dec 2, 4, 6) Chromatic quasisymmetric function fun
  - Homework 7 due Dec 6

## Classroom environment

In order to help make our classroom an excellent place to be in and learn mathematics, please keep in mind the following principles:

- ***Speaking up in class is encouraged!*** If you don't understand something, no matter how small, chances are someone else in the class doesn't understand it either, and asking will help me address the confusion and make things clearer. Off-the-wall ideas and comments are also always encouraged.

- ***Talk to each other!*** If I give a problem for you all to think about during class, chatting with your neighbor is encouraged; explaining helps both the explainer and the listener to understand better.
- ***Kindness:*** The students in this class will be coming from many different backgrounds, both mathematically and as human beings. Please be respectful and encouraging towards each other.

## **Academic Integrity**

This course will adhere to the CSU Academic Integrity Policy as found on the Student Responsibilities page of the CSU General Catalog and in the Student Conduct Code. At a minimum, violations will result in a grading penalty in this course and a report to the Office of Student Resolution Center.

## **Accommodations**

This is a disability-inclusive classroom. Students with disabilities who need accommodations can ask me directly, though I will ask you to contact the Student Disability Center (SDC) before approving accommodations such as extra time to finish homeworks. However, if there are other accommodations that are not related to course/grading policy that could help you participate more fully in class, please let me know.

More university resources are spelled out here: <https://col.st/2FA2g>