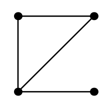
Math 502: Combinatorics
Homework 12

Recall that you must hand in a subset of the problems for which deleting any problem makes the total score less than 10. The maximum possible score on this homework is 10 points. See the syllabus for scoring details.

Problems

1. (1) [1 point] Show that the complex projective line $\mathbb{P}^1_{\mathbb{C}}$ is the same as the Riemann sphere.
2. (1+) [2 points] Compute the chromatic polynomial of the graph below using deletion and contraction:



3. (1+) [2 points] How many k-flats does $\mathbb{P}^{r-1}_{\mathbf{F}_1}$ have?
4. (1+) [2 points] How many SETs are there in the game of SET? In other words, how many lines are there in $\mathbb{F}_3^4$? (Warning: This is not the same as the number of 1-dimensional subspaces, since a line does not necessarily have to pass through the origin.)
5. (2+) [4 points] In Junior SET, the game is to find lines in $\mathbb{F}_3^3$ rather than $\mathbb{F}_3^4$. (The cards each have a color, number, and shape, but shading is no longer an attribute. Each card corresponds to a point of $\mathbb{F}_3^3$.) What is the probability that a given layout of 6 Junior SET cards chosen randomly from the deck of 27 cards does not contain any Junior SET (line)?
6. (1+) [2 points] How many Fourmations are there in the game of Fourmation? In other words, how many 2-planes are there in $\mathbb{F}_2^6$? (Warning: This is not the same as the number of 2-dimensional subspaces, since a line does not necessarily have to pass through the origin.)
7. (2-) [3 points] Prove that a projective transformation of $\mathbb{P}^1$ is uniquely determined by where it sends three points (say, $(0:1), (1:0), (1:1)$).
8. (3-) [8 points] Prove that all conics in the projective plane $\mathbb{P}^2_{\mathbb{R}}$ are equivalent up to a projective transformation. That is, prove that any two conics defined by quadratic equations of the form

$$ax^2 + by^2 + cz^2 + dxy + exz + fyz = 0$$

can be transformed into one another by applying a projective transformation to the homogeneous coordinates $(x : y : z)$.