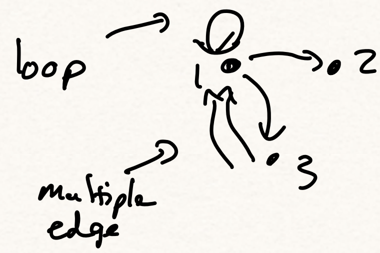
<u>Cayley's Thm</u>: n^{n-2} trees on n labeled vertices

<u>Def</u>: A <u>labeled</u> <u>directed</u> <u>graph</u> is a pair (V, E) where V is a set of <u>vertices</u> or <u>nodes</u> and $E \subseteq V \times V$ is a multiset of directed <u>edges</u>.

<u>Ex</u>: $V = \{1, 2, 3\}$, $E = \{(1,2), (1,3), (3,1), (3,1), (1,1)\}$



<u>Def</u>: <u>Undirected</u> graph: if $(a,b) \in E$ then $(b,a) \in E$, w/ same multiplicity. Consider $\{(a,b), (b,a)\}$ a single undirected edge.

<u>Def</u>: <u>Simple</u> <u>graph</u>: No loops or multiple edges, undirected.

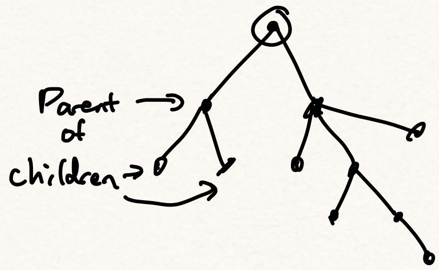
<u>Def</u>: <u>Cycle</u>: A sequence of edges $(v_1, v_2), (v_2, v_3), \ldots, (v_n, v_1)$

<u>Def</u>: <u>Forest</u>: A graph w/ no cycles.

<u>Def</u>: <u>Tree</u>: A <u>connected</u> graph w/ no cycles

paths btwn any two vertices

Def: **Rooted tree:** A tree w/ one vertex specified as the root.

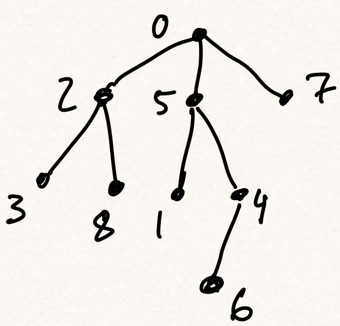


Note each rooted tree comes w/ natural orientations away from or towards the root.

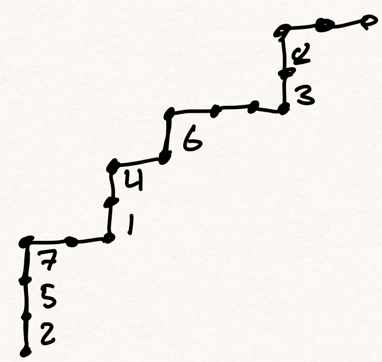
Thm (Cayley): # trees on vertex set $V = \{1, 2, 3, \ldots, n\}$ is n^{n-2}.

Pf: Want to give a bijection btwn trees on $0, 1, 2, \ldots, n$ and parking functions of size n.

Make 0 be the root, consider children: ordered L to R $\leq$



$\longrightarrow$



- Children of 0 is 1st col
- Starting from top of 1st col (7), put children in a col starting @ same diag as 7
- Inductively for grandchildren etc
- Do the same for 5, then 2

<u>Reverse bijection:</u> Make the first col be children, go diagonally from each until you either bump into a car or exit. If you bump into a car at the bottom of a column, that column is the granchildren, etc.